Problem 1

Evaluate $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$.

Solution

Make the substitution, $x=u^6$. Note that as $x\to 1$, $u\to 1$ as well.

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=\lim_{u\to 1}\frac{\sqrt[3]{u^6}-1}{\sqrt{u^6}-1}$$

$$=\lim_{u\to 1}\frac{u^2-1}{|u^3|-1}$$

The absolute value sign may be removed because u^3 is a positive number as $u\to 1$.

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=\lim_{u\to 1}\frac{u^2-1}{u^3-1}$$

$$=\lim_{u\to 1}\frac{(u+1)(u-1)}{(u-1)(u^2+u+1)}$$

$$=\lim_{u\to 1}\frac{u+1}{u^2+u+1}$$

$$=\frac{1+1}{1^2+1+1}$$

$$=\frac{2}{3}$$